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THE ENTRY OF A WEDGE INTO AN INCOMPRESSIBLE FLUID*

B.S. CHEKIN

The similarity problem of the entry of a rigid wedge into an ideal weightless incompressible fluid occupying a half-space is studied. The difficulty is that a non-linear boundary conditon has to be satisfied on the free surface of the fluid, whose position is unknown and has to be found during the solution. Three types of fluid motion are considered: flow past the wedge without break-away, the case when one wedge face is not wetted (a semi-infinite plate), and the intermediate case, when a cavity forms on one face. The problem amounts to solving a non-linear system of integral equations. A method of solving this system is given for the flow without break-away and the plate case. Examples of calculations are given. The results for thin and thick wedges are compared with approximate data.

The penetration of a wedge into a fluid was first studied in /1/. In /2/ the linear problem of normal collision with a water surface was solved. An approximate solution can be found e.g., in /3-5/. In /6/ a solution was obtained for the special case the entry of a wedge into a fluid. In /7/ the problem of normal wedge entry was solved in the exact non-linear statement, and the same problem was considered in /8/. The method below is based on that of /7/.

1. Let the wedge $M_1M_2M_3$ move with constant velocity $\mathbf{V}_w$ (Fig.1) and enter a fluid which occupies the lower half-space $Y \leqslant 0$ at the initial instant $t = 0$ and is at rest. At an instant $t > 0$ the distorted fluid boundary $N_1M_1M_2M_3N_3$ can have the shape shown in

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Fig.1. We call the sections N_1M_1, and M_3N_3 the free boundaries (FB), since we require that the pressure on them be zero. The sections M_1M_2, and M_2M_3 which are the same as the wedge faces will be called the unpenetrated boundaries (UB). We show in Fig.1 the angles α_1, α_3 and β, which characterize the wedge orientation in the space and the direction of the vector $\mathbf{V}_w$.

In Fig.1 we show only one of the possible types of fluid motion, that corresponds to a certain relation between α_1, α_3 and β, when there is no jet break-away from the rib M_2. If this relation does not hold, then break-away occurs, and there are two possible types of motion. One is shown in Fig.2, when one face is unwetted, which corresponds in essence to entry into the fluid of a semi-infinite plate or a wedge with a fairly small angle $\varkappa = \alpha_3 - \alpha_1$. The third type of motion is shown in Fig.3. A cavity is formed on one face. This type can be regarded as intermediate between the first two, and is obtained when the above relation between the angle α_1, α_2 and β, needed for there to be no break-away, is violated for fairly small variations of α_1, α_3 and β.

2. Assume that the fluid motion is potential. We can then introduce the harmonic function $\Phi(X, Y, t)$, which is the particle velocity potential $\mathbf{V} = \nabla\Phi$. The Lagrange integral gives the expression for the pressure $P = -\rho_0(\partial\Phi/\partial t + V^2/2)$, where ρ_0 is the density. We have a similarity problem. We introduce the dimensionless similarity coordinates $\mathbf{r} = \mathbf{R}/(V_w t)$, where R is the position vector with the components X, Y. Denote the components of the vector $\mathbf{r}$ by x, y. We introduce the dimensionless velocity potential $\psi(x, y) = \Phi/(tV_w^2)$, the velocity $\mathbf{v} = \mathbf{V}/V_w$, and the pressure $p = P/(\rho_0 V_w^2)$. Then, $\mathbf{v} = \nabla\psi$. In the new variables the Lagrange integral, and the kinematic condition, which amounts to requiring that the FB consists of the same particles, can be written as

$$p = \mathbf{v}\mathbf{r} - {}^1/_2 v^2 - \psi \tag{2.1}$$

$$\boldsymbol{\nu}(\mathbf{v} - \mathbf{r}) = 0 \tag{2.2}$$

where $\boldsymbol{\nu}$ is the normal to the boundary. Since we must have $\mathbf{v}\boldsymbol{\nu} = \mathbf{r}_2\boldsymbol{\nu} = \mathbf{r}\boldsymbol{\nu}$, on UB, where $\mathbf{r}_2$ corresponds to the point M_2, then Eq.(2.2) holds, not only on FB, but also on the entire fluid boundary.

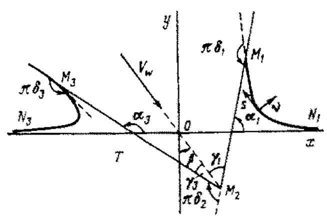

Fig.1

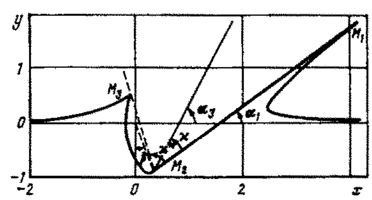

Fig.2

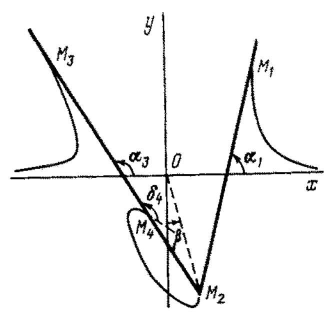

Fig.3

Let T be the domain occupied by the fluid in the xy plane, Fig.1, and s the boundary arc length, measured from a fixed point. We assume that s is increasing if, as we move along the boundary, the domain T is circuited counterclockwise. Let $\mathbf{s}$ be the unit vector tangential to the boundary. Then, $\mathbf{s} = d\mathbf{r}/ds = \mathbf{r}'(s)$, where $\mathbf{r}(s)$ is the position vector of the boundary point. Differentiating (2.1) with respect to s, we obtain on the FB, noting that $p'(s) = 0$,

$$\mathbf{v}'(s)(\mathbf{r} - \mathbf{v}) = 0 \tag{2.3}$$

We can write (2.2) and (2.3) as the single vector expression

$$\mathbf{v} = \mathbf{r} - (s + s_0)\,\mathbf{s}, \quad s_0 = \mathbf{s}_0(\mathbf{r}_0 - \mathbf{v}_0) \tag{2.4}$$

where $\mathbf{r}_0$ corresponds to a fixed point on the FB. It is clear from (2.2) and (2.3), that the vector **r** - **v** is directed along **s**, while $\mathbf{v}'(s)$ is orthogonal to **s** on the FB and is directed along **s** on the UB.

3. We introduce the complex plane $z = x + iy$ and the complex velocity $V = v_x - iv_y$. Following /7/, we also introduce a complex plane $w = u + i\bar{v}$. Let $w = w(z)$ map conformally the domain T into the upper half-plane $\operatorname{Im} w > 0$. The boundary of T maps one-to-one onto the real axis $\operatorname{Im} w = 0$. By Riemann's theorem, this mapping exists. The function $z(w)$ maps conformally the half-plane $\operatorname{Im} w > 0$ into the domain T. Under this mapping, boundary points M_i $(i = 1, 2, \ldots)$ become points u_i on the real axis $\operatorname{Im} w = 0$, two of which can be chosen arbitrarily.

We write the function $z'(w)$ as

$$z'(w) = a\omega(w)\,e^{i\vartheta(w)}; \quad a = \text{const} > 0, \quad \omega(w) > 0$$

where $\vartheta(u)$ is the angle between the x axis and the vector **s**, measured counterclockwise from the x axis. We also introduce the angle $\theta(u) = \pi - \vartheta(u)$. Obviously, we must have $\theta(u) \to 0$ as $|u| \to \infty$. Since the function

$$i\ln\left(e^{-i\pi}z'(w)\right) = i\ln\left(a\omega(w)\right) + \theta(w)$$

is analytic in the half-plane $\operatorname{Im} w > 0$ and takes real values $\theta(u)$ on the axis $\operatorname{Im} w = 0$, then $z'(w)$ can be written in terms of $\theta(u)$:

$$z'(w) = -a\exp\left(-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\theta(u)}{u - w}\,du\right)$$

Letting $w \to u$ and using Plehmel's relation, we have

$$z'(u) = -a\omega(u)\,e^{-i\theta(u)}, \quad \omega(u) = \exp\left(-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\theta(t)\,dt}{t - u}\right) \tag{3.1}$$

The integral in (3.1) has to be understood in the sense of the principal value. When obtaining (3.1), it was assumed that $\theta(u)$ is piecewise continuous and bounded, and satisfies Hölder's condition on every part of the u axis where it is continuous. The function $\omega(u) \to 1$ as $|u| \to \infty$, and can either vanish or become infinite as $u \to u_i$. These singularities have the form

$$\omega(u) \to \text{const}\,|u - u_i|^{-\delta_i}, \quad u \to u_i$$

where $\pi\delta_i$ is the angle through which the vector **s** rotates in the neighbourhood of the point M_i (Fig.1), where $\delta_i > 0$, if **s** rotates counterclockwise at a boundary break point. The vector **s** has the components $s_x = -\cos\theta$, $s_y = \sin\theta$.

For the complex coordinate of the boundary we have

$$z(u) = -a\int_{u_2}^{u}\omega(t)\,e^{-i\theta(t)}dt + z_2, \quad z_2 = \sin\beta - i\cos\beta$$

where the angle θ is known on the pieces of the UB.

We introduce the function, analytic in the half-plane $\operatorname{Im} w > 0$:

$$\chi(w) = z'(w)\,V'(w)$$

Hence

$$V(w) = \int_{\infty}^{w}\chi(\tau)\,\frac{d\tau}{z'(\tau)} \tag{3.2}$$

On the real axis $\operatorname{Im} w = 0$ we have

$$V(u) = -\frac{1}{a}\int_{-\infty}^{u}\frac{\chi(t)}{\omega(t)}\,e^{i\theta(t)}dt \tag{3.3}$$

From (3.3) we have

$$\chi(u)_{-} = a\omega(u)\,\mathbf{v}'(u)(\mathbf{s} + i\mathbf{v})$$

It follows from this relation that the function $\chi(u)$ is real on the UB and imaginary on the FB. Using (3.3), it can be shown that condition (2.2) holds on each piece of the UB if we require that it holds at just one point of the piece.

Apart from (3.3) for the velocity on the FB, we also have (2.4). On writing (2.4) in complex form and equating the right-hand side to the right-hand side of (3.3), we find that, on each piece of the FB.

$$\frac{1}{a}\int\limits_{-\infty}^{u}\frac{\chi(t)}{\omega(t)}e^{i\theta(t)}dt + \bar{z}(u) + \left[a\int\limits_{u_0}^{u}\omega(t)\,dt + s_0\right]e^{i\theta(u)} = 0 \tag{3.4}$$

(the bar over z denotes the complex conjugate quantity and u_0 corresponds to a fixed point on the piece of the FB). Differentiating (3.4) with respect to u, on each piece of the FB where the angle $\theta(u)$ is continuous, we obtain

$$\theta'(u) = i\chi(u)\left\{a\omega(u)\left[\int\limits_{u_0}^{u}\omega(t)\,dt + s_0\right]\right\}^{-1} \tag{3.5}$$

It can be shown that Eq.(3.5) is equivalent to (3.4), if we require that condition (2.2) hold at just one point on the given piece of the FB, e.g., at the point $u = u_0$:

$$\mathbf{v}(u_0)\left[\mathbf{v}(u_0) - \mathbf{r}(u_0)\right] = 0 \tag{3.6}$$

Thus, on any piece of the boundary, whether the FB or the UB, condition (3.6) must hold at a point $u = u_0$. Condition (2.2) must then hold on the entire boundary and

$$p'(u) = 0 \tag{3.7}$$

on any piece of the FB.

For all three types of motions *1, 2, 3* (Figs.1, 2, 3, respectively), we require that, on the free fluid surfaces, we have

$$p(u) = 0 \text{ on FB} \tag{3.8}$$

We write the pressure as

$$p(w) = \operatorname{Re}\varphi(w) - {}^1\!/_2 V(w)\bar{V}(w) \tag{3.9}$$

$$\varphi(w) = z(w)V(w) - \int\limits_{\infty}^{w}V(\tau)z'(\tau)\,d\tau$$

It follows from (3.2), (3.7) and (3.9) that the functions $V(w)$, $\varphi(w)$, analytic in the upper half-plane $\operatorname{Im} w > 0$, vanish as $w \to \infty$, and condition (3.8) holds automatically for the types of motion *1* and *2*. In the case of type *3*, condition (3.8) will also hold provided that we require that the pressure be zero at a point on the free boundary M_2M_4 (Fig.3), e.g., at the point M_2

$$p(u_2) = 0 \tag{3.10}$$

A further condition must hold for type 3: the point M_4 must actually lie in the face M_2M_3. This condition can be written as ($\mathbf{v}_{23}$ is normal to face M_2M_3)

$$\mathbf{v}_{23}\int\limits_{u_2}^{u_4}\omega(t)\,\mathbf{s}(t)\,dt = 0 \tag{3.11}$$

Notice that, by relations (3.4) and (3.6) and the continuity of $V(u)$, we have $y(u) \to 0$ as $|u| \to \infty$ and $\mathbf{v}_i = \mathbf{r}_i$ at boundary break points, where $\delta_i \neq 0$, e.g., at points M_1, M_3.

4. The function $\chi(w)$, which is analytic in the half-plane $\operatorname{Im} w > 0$, and takes real values on the pieces of real axis $\operatorname{Im} w = 0$ that correspond to the UB, and imaginary values on the pieces corresponding to the FB, can be found for our three types of motion. When obtaining $\chi(w)$ we took account of the following: the velocity $V(u)$ is continuous, the function on the right of (3.5), and also the function $\omega(u)$, $\chi(u)/\omega(u)$ are integrable, $\theta(u) \to 0$ as $|u| \to \infty$, the pressure $p(w) \to 0$ as $w \to \infty$, the pressure on the UB is not negative, and any two boundary break points cannot be ends of the same piece of the FB.

This follows from the equation v r at these points, and relation (2.4).
We find:
for type 1

$$\chi(w) = -ib(w-u_1)^{-3/2}(w-u_3)^{-3/2}$$

for type 2

$$\chi(w) = -ib(w-u_1)^{-3/2}(w-u_2)^{-1/2}(w-u_3)^{-1}$$

where, in this case (Fig.2) it turns out that the angle $\theta(u)$ is continuous at the point M_2 and the FB must necessarily have a break at a point M_3 ($\delta_3 > 0$);
for type 3

$$\chi(w) = -ib[(w-u_1)(w-u_4)(w-u_3)]^{-3/2}(w-u_2)^{-1/2} \times (b_0 + b_1 w + w^2)$$

where b, b_0, b_1 are constants. As in the case of type 2, there cannot be a boundary break at the point M_2.

Cuts of $\chi(w)$, analytic in the half-plane $\operatorname{Im} w > 0$ must be located in the half-plane $\operatorname{Im} w < 0$, and when $u \gg u_3$, we put $\operatorname{Im}\chi(u) < 0$.

Since the above-mentioned functions are integrable, we have the following bounds on the wanted angle $\theta(u)$:

$$3\pi/4 - \alpha_1 < \theta(u_1 - 0) < \pi - \alpha_1, \quad ^3/_4 < \delta_1 < 1 \quad \text{(types 1, 2, 3)}$$
$$-\alpha_3 < \theta(u_3 + 0) < \pi/4 - \alpha_3, \quad ^3/_4 < \delta_3 < 1 \quad \text{(types 1, 3)}$$
$$\pi/2 < \theta(u_3 - 0) - \theta(u_3 + 0) < \pi, \quad ^1/_2 < \delta_3 < 1 \quad \text{(type 2)}$$
$$3\pi/4 - \alpha_3 < \theta(u_4 - 0) - \pi < \pi - \alpha_3, \quad ^3/_4 < \delta_4 < 1 \quad \text{(type 3)}$$

It can be shown that the pieces of the FB that depart to infinity cannot have points of inflection, and lie entirely in the half-plane $y > 0$. We have the asymptotic relations as $|u| \to \infty$

$$\theta(u) \to -b/(3a^2u^3), \quad y(u) \to b/(6au^2)$$
$$v_x(u) \to -b^2/(15a^3u^5), \quad v_y(u) \to b/(2au^2)$$

In these relations the parameter u can be replaced by $-x/a$.

5. Consider the types of motion 1, 2, 3. We use the notation

$$c = b/a^2, \quad \xi(u) = |\chi(u)|/b, \quad \eta(u) = \operatorname{sign}(b_0 + b_1 u + u^2)$$
$$\Omega_k(u) = \omega(u)\int_{v_k}^{u}\omega(t)\,dt, \quad I_k(\alpha,\beta) = -c\int_{\alpha}^{\beta}\frac{\xi(t)}{\Omega_k(t)}\,dt$$
$$J_k(\alpha,\beta) = -c\int_{\alpha}^{\beta}\frac{\xi(t)\eta(t)}{\Omega_k(t)}\,dt, \quad k = 1,3,4$$

The function $\omega(u)$ must be expressible in terms of $\theta(u)$ by (3.1).

Type 1. We write Eqs.(3.5) as the system of integral equations

$$\theta(u) = I_1(-\infty, u),\ u \leqslant u_1;\ \theta(u) = I_3(u, \infty),\ u \geqslant u_3 \tag{5.1}$$

Let $u_1 = -1, u_3 = 1$. We then have four relations of the type (3.6), in which, e.g., we put $u_0 = u_1 - 0$, $u_0 = u_1 + 0$, $u_0 = u_3 - 0$, $u_0 = u_3 + 0$. The resulting equations can be used to find u_2, a, b, and a relation between the angles $\alpha_1, \alpha_3, \beta$. For instance, if we fix α_1, α_3, we find from these equations u_2, a, b, and the angle β at which the type of motion shown in Fig.1 is possible, when there is no jet break-away from the wedge rib.

Type 2. The angle $\theta(u)$ satisfies on the three parts of the FB (Fig.2) the equations

$$\theta(u) = I_1(-\infty, u),\ u \leqslant u_1;\ \theta(u) = I_3(u_2, u) - \alpha_1,\ u_2 \leqslant u \leqslant u_3$$
$$\theta(u) = I_3(u, \infty),\quad u \geqslant u_3$$

Let $u_1 = -1, u_3 = 1$. Noting that the angle $\theta(u)$ is continuous at the point $u = u_2$, it can be shown that there are then only three independent equations of type (3.6). These equations can be used to find u_2, a and b.

Type 3. For the angle $\theta(u)$ on the three pieces of FB (Fig.3) we obtain

$$\theta(u) = J_1(-\infty, u),\ u \leqslant u_1;\ \theta(u) = J_4(u_2, u) - \alpha_1,\ u_2 \leqslant u \leqslant u_4$$
$$\theta(u) = J_3(u, \infty),\ u \geqslant u_3$$

We put $u_1 = -1$, $u_3 = 1$. For the six unknown constants u_2, u_4, a, b, b_0, b_1 we have six equations: 4 independent Eqs.(3.6) (note that the angle $\theta(u)$ is continuous at the point M_2), and (3.10), (3.11).

These equations involve jet break-away from the left face of the wedge. If the relations between the angles $\alpha_1, \alpha_3, \beta$ correspond to jet break-away from the right face, we can use our expressions by replacing these angles by $\pi - \alpha_3$, $\pi - \alpha_1$ and $-\beta$ respectively.

6. We will suggest an interative process for solving these equations in the case of motions of types *1* and *2*. We regard the solution as being obtained if the process converges when realized numerically. We will describe the method for the simple example when the wedge enters the fluid normally, and $\alpha_1 + \alpha_3 = \pi$, $\beta = 0$. We can then put $u_3 = 1$, $u_1 = -1$, $u_2 = 0$, and there are only two independent equations of type (3.6) (from which a and b are found), while instead of system (5.1) we have the single equation

$$\theta(u) = c(1-\delta)\int_0^\infty \left(\frac{t-1}{t}\right)^{2\delta - {}^5/_2} G(t)\frac{dt}{t^4},\quad u \geqslant 1 \tag{6.1}$$
$$\pi\delta = \pi/2 - \alpha + \theta_0,\quad \theta_0 = \theta(u_3 + 0),\quad 3/4 < \delta < 1$$

where $G(u)$ is a continuous function, which depends on $\theta(u)$, and α is half the angle of the wedge. We isolate the singularity in the integrand on the right of (6.1), the nature of which depends on the solution (depends on θ_0).

Let the n-th approximation $\theta_n(u)$ be known. The $(n+1)$-th approximation is then found as follows. The quantities $G(u)$ and c are calculated from the n-th approximation, while $\theta_{n+1}(u)$ is found by solving the equation

$$\theta_{n+1}(u) = c_n(1-\delta_{n+1})\int_u^\infty \left(\frac{t-1}{t}\right)^{2\delta_{n+1} - {}^5/_2} G_n(t)\frac{dt}{t^4} \tag{6.2}$$

Putting $u = 1$ in (6.2), we arrive at the equation for $\theta_{n+1}(1)$. Its solution, which satisfies the necessary inequality $\pi/4 + \alpha < \theta_{n+1}(1) < \pi/2 + \alpha$, exists. Having found $\theta_{n+1}(1)$ and hence δ_{n+1}, we obtain $\theta_{n+1}(u)$ by integration from (6.2). The first approximation $\theta_1(u)$ can be found by putting $G(u) = 1$ in (6.1).

In the case of motion of type *3*, it is difficult to use this method, since it is not possible to prove that the equation of type (6.2) with $u = 1$ has a solution that satisfies the appropriate inequalities.

7. To sum up, there are three types of motion. It has been shown above that motion without break-away (type *1*) can only exist when there is a certain relation between the angles α_1, α_3, and β. We can always write this relation as

$$\gamma_3/\gamma_1 = f(\varkappa, \beta) \tag{7.1}$$
$$\gamma_1 = \pi/2 - \alpha_1 + \beta,\ \gamma_3 = \alpha_3 - \pi/2 - \beta,\ \varkappa = \gamma_1 + \gamma_3$$

The angles γ_1, γ_3 are shown in Fig.1. The line OM_2, which coincides with the wedge velocity V_w, must divide the wedge angle $\varkappa$ into two parts γ_1 and γ_3, such that there is no jet break-away from the rib.

In the case of normal wedge entry $(\beta = 0)$ we have $\gamma_1 = \gamma_3$, and $f(\varkappa, 0) = 1$. In general, $f(\varkappa, \beta)$ is now known in advance and its value for the given $\varkappa, \beta$ can be found only while solving Problem 1 (type *1*). On fixing $\varkappa$ and performing calculations for different β, we can plot $f(\varkappa, \beta)$, i.e., a curve in the $(\gamma_3/\gamma_1, \beta)$ plane.

We show such a curve *1* in Fig.4 for $\varkappa = 0.2\pi$, with $0 \leqslant \beta \leqslant \pi/2$. The points of the $(\gamma_3/\gamma_1, \beta/\pi)$ plane which lie below this curve correspond to jet break-away from the left face, while points above the curve correspond to jet break-away from the right face. If these points are sufficiently close to the curve (7.1), we have motion of type 3. Curve 2 in Fig.4 is plotted for $\varkappa = 0.04\pi$, and curve 3, for an infinitely small angle $\varkappa$. This last curve was obtained from approximate linear theory, which takes no account of the lift of the free fluid surface.

In the case of break-away motion, it is again impossible to indicate in advance the limits to be imposed on the angles $\alpha_1, \alpha_3, \beta$, such that either type 2 or type 3 motion is seen. These limits can only be found while solving the problem. Given any $\alpha_1, \alpha_3, \beta$, we can always find the type of motion that corresponds to these angles. Fix α_1, β. For a sufficiently small wedge angle $\varkappa$, there is a type 2 motion. On solving Problem 2, we find the wedge angle $\varkappa = \varkappa^*$ for which the point M_3 lies on a face (Fig.2). With $0 \leqslant \varkappa < \varkappa^*$, we have type 2. Solving Problem 1, we find the angle $\varkappa = \bar{\varkappa}$, that corresponds to motion without break-away (type 1). With $\varkappa^* \leqslant \varkappa < \bar{\varkappa}$ we have type 3. With $\varkappa > \bar{\varkappa}$, there is motion with jet break-away, but from the other face.

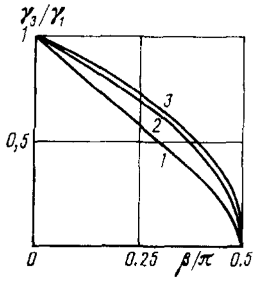

Fig.4

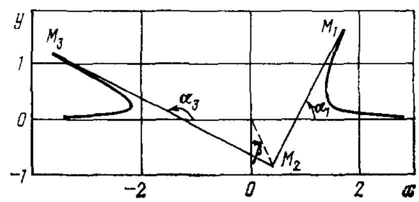

Fig.5

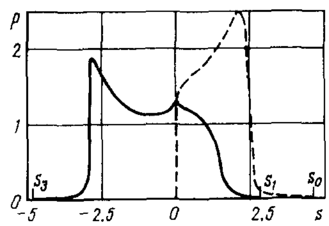

Fig.6

Consider a specific example. Let $\alpha_1 = 0.6\ \pi$, $\beta = 0.25\pi$. Calculations give $\varkappa^* = 0.131\pi$, $\bar{\varkappa} = 0.235\pi$. Thus, with $0 \leqslant \varkappa < 0.131\pi$, $0.131\pi \leqslant \varkappa < 0.235\pi$, $\varkappa = 0.235\pi$, we have types 2, 3, 1 respectively.

8. Let us give some theoretical results for motions of types 1 and 2. Here, s is the distance along the face from M_2; $s > 0$ refers to the right face, and $s < 0$ to the left face. Table 1 gives, for normal wedge entry, the dependence of the force F on the wedge half-angle α:

$$F = \int_0^{s_1} p(s)\, ds\ ,$$

where F acts on a face, the pressure on the wedge rib is $p_w = p\ (0)$, and the angle $\rho\pi = \pi - \pi\delta_1$.

In /8/, where the calculations were made for $\alpha = \pi/4$, the quoted data are $\rho = 0.020$, $F_y = 2F \sin \alpha = 3.40$. From Table 1: $\rho = 0.021$, and $F_y = 3.45$.

Given sufficiently large α, the maximum pressure p_m is greater than p_w. For instance, with $\alpha = 0.4\pi$, we have $p_w = 4.24$, $p_m = p\ (4.92) = 11.2$. In the last column of Table 1 we give the force $F = F_0$, calculated from the approximate expressions. The first four rows of this column refer to a thin wedge, and the last to the results calculated from the expression in /1/ for a blunt wedge, allowing for the lift of the free surface. It can be seen from Table 1 that, for $\alpha \ll 1$, the pressure p_w is well described by $p_w = 0.5 + 2 \ln 2 \cdot \alpha/\pi$, which was obtained in /5/ for a blunt wedge.

The disturbed fluid surface shape is given in Fig.5 for $\alpha_1 = 0.35\pi$, $\alpha_3 = 0.85\pi$. The angle at which there is no jet break-away from the rib is $\beta = 0.153\pi$. For the angles which define the surface breaks at the points M_1, M_3, we have $\pi\ (1 - \delta_1) = 0.0358\pi$, $\pi\ (1 - \delta_3) = 0.0083\pi$.

The continuous curve of Fig.6 is the pressure distribution on the faces M_2M_1, M_2M_3. The forces on these faces are

$$F_1 = \int_0^{s_1} p(s)\,ds = 1.55, \quad F_3 = \int_{s_3}^{0} p(s)\,ds = 3.62$$

The modulus of the derivative $|p'(s)|$ increases without limit as we approach the rib. In general, as $s \to 0$, we have $|p'(s)| \to \infty$ for a wedge angle $\varkappa < 2\pi/3$, and $p'(s) \to 0$ for $\varkappa > 2\pi/3$.

Table 1

α/π	F	p_w	ρ	F_*
0.001	$0.140 \cdot 10^{-2}$	0.501	0.102	$0.139 \cdot 10^{-2}$
0.005	$0.725 \cdot 10^{-2}$	0.507	$0.964 \cdot 10^{-1}$	$0.693 \cdot 10^{-2}$
0.01	$0.151 \cdot 10^{-1}$	0.514	$0.926 \cdot 10^{-1}$	$0.139 \cdot 10^{-1}$
0.05	0.103	0.581	$0.715 \cdot 10^{-1}$	$0.693 \cdot 10^{-1}$
0.1	0.289	0.688	$0.536 \cdot 10^{-1}$	
0.2	1.26	1.05	$0.292 \cdot 10^{-1}$	
0.3	4.86	1.82	$0.138 \cdot 10^{-1}$	
0.35	10.62	2.61	$0.820 \cdot 10^{-2}$	
0.4	28.8	4.24	$0.400 \cdot 10^{-2}$	
0.45	136	9.1	$0.110 \cdot 10^{-2}$	156

Consider a plate with $\alpha_1 = 0.2\pi$, $\beta = 0.1\pi$. In Fig.2 we show the fluid surface shape; the broken curve in Fig.6 is the pressure distribution $p(s)$ along the plate. The force acting on the plate is

$$F = \int_0^{s_0} p(s)\,ds = 4.64$$

At the point $s = 0$ the derivative $p'(s)$ has a singularity of the type $1/\sqrt{s}$.

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